Cambridge O Level

Cambridge International Examinations
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **May/June 2016**

MARK SCHEME

Maximum Mark: 80

Published

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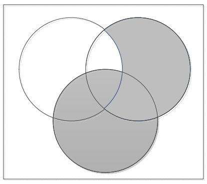
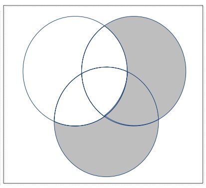
Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

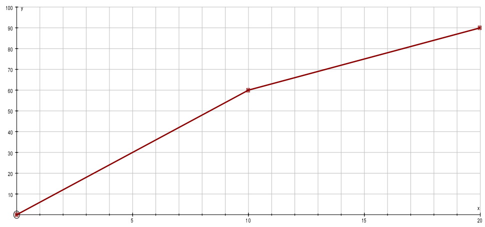
Question	Answer	Marks	Guidance
1 **(a)**	$Y \subset X$ or $Y \subseteq X$ only $Y \cap Z = \varnothing$ or $\{\ \}$ only	**B1** **B1**	
(b)	**(i)**  **(ii)** 	**B1** **B1**	
2 **(i)**	$32 - \frac{20}{x} + \frac{5}{x^2}$	**B3**	**B1** for each correct term – must be integers
(ii)	$(3\times 32) + \left(-\frac{20}{x}\times 4x\right) = 16$ Accept $16x^{\text{o}}$	**M1** **A1**	for $(3\times \textit{their}\ 32) + \left(\frac{\textit{their}(-20)}{x}\times 4x\right)$
3 **(i)**	$\mathbf{b}-\mathbf{c} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ $4 + y^2 = 36 + 4$ $y = \pm 6$	**B1** **M1** **A1**	may be implied by further correct working for one correct attempt at using the modulus
(ii)	$\mu + 4 = 2\lambda$ or $-4\mu + 24 = 7\lambda$ $\mu - 4 = -\lambda$ or $8\mu - 8 = \lambda$ leading to $\mu = \frac{4}{3}$, $\lambda = \frac{8}{3}$ oe allow 1.33 and 2.67 or better	**B1** **B1** **DB1**	for one correct equation in μ and λ for a second correct equation in μ and λ for both, must have both previous **B** marks

Question	Answer	Marks	Guidance
4	$\left(4+\sqrt{5}\right)x^2+\left(2-\sqrt{5}\right)x-1=0$		**You must be convinced that a calculator is not being used.**
	$x=\dfrac{-\left(2-\sqrt{5}\right)\pm\sqrt{\left(2-\sqrt{5}\right)^2-4\left(4+\sqrt{5}\right)(-1)}}{2\left(4+\sqrt{5}\right)}$	**M1** **A1**	for use of quadratic formula (allow one sign error), allow $b^2=9-4\sqrt{5}$ all correct
	$x=\dfrac{-\left(2-\sqrt{5}\right)\pm\sqrt{9-4\sqrt{5}+16+4\sqrt{5}}}{2\left(4+\sqrt{5}\right)}$	**DM1**	for attempt to simplify the discriminant (minimum of 4 terms must be seen in discriminant, 2 terms involving √5 and 2 constant terms)
	$=\dfrac{-\left(2-\sqrt{5}\right)+5}{2\left(4+\sqrt{5}\right)}$		
	$=\dfrac{3+\sqrt{5}}{2\left(4+\sqrt{5}\right)}$	**A1**	for $\dfrac{3+\sqrt{5}}{2\left(4+\sqrt{5}\right)}$ or $\dfrac{3+\sqrt{5}}{8+2\sqrt{5}}$, ignore negative solution if included
	$=\dfrac{\left(3+\sqrt{5}\right)\left(4-\sqrt{5}\right)}{2\left(4+\sqrt{5}\right)\left(4-\sqrt{5}\right)}$	**M1**	for attempt to rationalise an expression of the form $\dfrac{a\pm b\sqrt{5}}{c\pm d\sqrt{5}}$ as part of their solution of the quadratic Must obtain an integer denominator
	$=\dfrac{7+\sqrt{5}}{22}$	**A1**	**Final A1 can only be awarded if all previous marks have been obtained**
5 (i)	$\left(1-\cos\theta\right)\left(1+\sec\theta\right)$ $=1-\cos\theta+\dfrac{1}{\cos\theta}-\dfrac{\cos\theta}{\cos\theta}$ $=\sec\theta-\cos\theta$ $=\dfrac{1}{\cos\theta}-\cos\theta$	**M1**	M1 for expansion and use of $\sec\theta=\dfrac{1}{\cos\theta}$ consistently, allow one sign error
	$=\dfrac{1-\cos^2\theta}{\cos\theta}$	**DM1**	for attempt at a single fraction, dependent on first **M1**
	$=\dfrac{\sin^2\theta}{\cos\theta}$	**A1**	
	$=\sin\theta\tan\theta$ www	**A1**	

Question	Answer	Marks	Guidance
	Alternative method:		
	$(1-\cos\theta)\left(\dfrac{\cos\theta+1}{\cos\theta}\right)$	M1	for attempt at a single fraction for second factor and use of $\sec\theta=\dfrac{1}{\cos\theta}$
	$=\dfrac{1-\cos^2\theta}{\cos\theta}$	DM1	for expansion
	$=\dfrac{\sin^2\theta}{\cos\theta}$	A1	
	$=\sin\theta\tan\theta$ www	A1	
(ii)	$\sin\theta\tan\theta=\sin\theta$ $\sin\theta(\tan\theta-1)=0$		
	$\tan\theta=1,\ \theta=\dfrac{\pi}{4}$, allow 0.785 or better	B1	for $\theta=\dfrac{\pi}{4}$ from $\tan\theta=1$
	$\sin\theta=0,\ \theta=0,\ \pi$ or 3.14 or better	B1 B1	for $\theta=0$ from $\sin\theta=0$ for $\theta=\pi$ from $\sin\theta=0$
6	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x}(4x+1)^{\frac{1}{2}}\right)$		
	$=\mathrm{e}^{3x}\dfrac{1}{2}\times4(4x+1)^{-\frac{1}{2}}+3\mathrm{e}^{3x}(4x+1)^{\frac{1}{2}}$	B1	for $r\mathrm{e}^{3x}(4x+1)^{-\frac{1}{2}}$ must be part of a sum, $r=\dfrac{1}{2}$ or 2 or $\dfrac{1}{2}\times4$
		B1	for $s\mathrm{e}^{3x}(4x+1)^{\frac{1}{2}}$ must be part of a sum, s is 1 or 3
	$=\dfrac{2\mathrm{e}^{3x}}{(4x+1)^{\frac{1}{2}}}+3\mathrm{e}^{3x}(4x+1)^{\frac{1}{2}}$	B1	for all correct, allow unsimplified
	$=\dfrac{\mathrm{e}^{3x}}{(4x+1)^{\frac{1}{2}}}(2+12x+3)$	DM1	for $\dfrac{\mathrm{e}^{3x}}{(4x+1)^{\frac{1}{2}}}(a+bx)$, dependent on first 2 **B** marks , must be using a correct method, collecting terms in the numerator correctly
	$=\dfrac{\mathrm{e}^{3x}}{(4x+1)^{\frac{1}{2}}}(12x+5)$	A1	
7 (i)	$\cos3x=\dfrac{1}{2},\quad x=\dfrac{\pi}{9}$ or $0.349,\ 20^{\circ}$, allow 0.35	M1 A1	for correct attempt to solve the trigonometric equation
(ii)	$B\left(\dfrac{\pi}{3},\ 3\right)$ or $(1.05,\ 3)$, $(60^{\circ},\ 3)$	B1B1	**B1** for each, must be in correct position or in terms of $x=$ and $y=$

Question	Answer	Marks	Guidance
(iii)	$\int_{\frac{\pi}{9}}^{\frac{\pi}{3}} 1-2\cos 3x \, dx = \left[x - \frac{2}{3}\sin 3x\right]_{\frac{\pi}{9}}^{\frac{\pi}{3}}$	M1 A1	for $x \pm a\sin 3x$ attempt to integrate at least one term for correct integration
	$= \frac{\pi}{3} - \left(\frac{\pi}{9} - \left(\frac{2}{3} \times \frac{\sqrt{3}}{2}\right)\right)$	DM1	for correct use of limits from **(i)** and **(ii)**, must be in radians
	$= \frac{2\pi}{9} + \frac{\sqrt{3}}{3}$ oe or 1.28	A1	
8 (i)	$\lg y = x^2 \lg b + \lg A$ $\lg b = \pm 0.21$ $b = 0.617$ allow 0.62, $10^{-0.21}$ $\lg A = 0.94$ allow 0.93 to 0.95 $A = 8.71$ allow awrt 8.5 to 8.9	 B1 B1 B1 B1	for $\lg b = \pm 0.21$ may be implied
	Alternative method 5.37 or $10^{0.73} = Ab$ 1.259 or $10^{0.1} = Ab^4$ $b^3 = 10^{-0.63}$ $b = 0.617$ allow 0.62, $10^{-0.21}$ $A = 8.71$ allow awrt 8.5 to 8.9	 B1 B1 B1 B1	for both equations, allow correct to 2 sf
(ii)	$x = 1.5$, $x^2 = 2.25$ $y = 2.93$, allow awrt 2.9 or 3.0	M1 A1	for correct use of graph $y = theirA \times theirb^{1.5^2}$ or $\lg y = \lg theirA + \left(1.5^2 \lg theirb\right)$
(iii)	$\lg y = 0.301$, or $2 = \text{'}8.71(0.617)^{x^2}\text{'}$ $x = 1.74$, allow $\sqrt{3}$ or awrt 1.7, 1.8	M1 A1	for correct use of graph to read off x^2 $2 = theirA(theirb)^{x^2}$ or $\lg 2 = (\lg theirb)x^2 + \lg(theirA)$
9 (i)	$y = \frac{2}{3}(3x+10)^{\frac{1}{2}}$ $(+c)$	B1 B1	for $p(3x+10)^{\frac{1}{2}}$ where p is a constant for $\frac{2}{3}(3x+10)^{\frac{1}{2}}$ oe unsimplified
	passes through $\left(2, -\frac{4}{3}\right)$, so $c = -4$	M1	for attempt to find c, must have attempt to integrate, must have the first **B1**
	$y = \frac{2}{3}(3x+10)^{\frac{1}{2}} - 4$ oe	A1	

Question	Answer	Marks	Guidance
(ii)	When $x = 5$, $y = -\frac{2}{3}$	B1	
	perpendicular gradient $= -5$	B1	
		M1	for attempt at the normal using *their* perpendicular gradient and *their* *y* value (but not $y = -\frac{4}{3}$ or $-\frac{5}{3}$).
	Equation of normal: $y + \frac{2}{3} = -5(x-5)$	A1	
	When $y = -\frac{5}{3}$,	DM1	for use of $y = -\frac{5}{3}$ in their normal equation to get as far as $x = ...$
	$x = 5.2$ oe	A1	
10 (i)	Area: $20 = \pi x^2 + xy$	B1	
	$y = \frac{20 - \pi x^2}{x}$	B1	
	$P = 2\pi x + 2x + 2y$ $= 2\pi x + 2x + 2\left(\frac{20}{x} - \pi x\right)$	M1	for attempt to use perimeter and obtain in terms of *x* only
	$= 2x + \frac{40}{x}$	A1	all steps seen, www AG
	Alternative method: $20 = \pi x^2 + xy$	B1	
	$P = 2\pi x + 2y + 2x$ $= \frac{2}{x}\left(\pi x^2 + xy\right) + 2x$	M1	for attempt to use perimeter and write in $\frac{\pi x^2 + xy}{x}$
	$= \frac{2}{x}(20)$ $+2x$	B1	for replacing $\pi x^2 + xy$ with 20
	$= 2x + \frac{40}{x}$	A1	all steps seen, www AG

Question	Answer	Marks	Guidance
(ii)	$\frac{dP}{dx} = 2 - \frac{40}{x^2}$	M1	for attempt to differentiate
	When $\frac{dP}{dx} = 0$,	DM1	for equating to zero and attempt to solve at least as far as $x^2 =$
	$x = 2\sqrt{5}$ allow 4.47, $\sqrt{20}$	A1	
	leading to $P = 8\sqrt{5}$, allow 17.9	A1	
	$\frac{d^2P}{dx^2} = \frac{80}{x^3}$, always positive so a minimum	A1	for this statement or use of gradient inspection either side of correct *x*
11 (a) (i)	Distance = area under graph	M1	for attempt to find the area, one correct area seen (triangle, rectangle or trapezium) as part of a sum.
	= 1275	A1	
(ii)	deceleration is 1.5 oe	B1	
(b)		B1	for a straight line between $(0,0)$ and $(10,60)$
		B1FT	**FT** a straight line between $(10, 60)$ and $(20, 90)$, a displacement vector $\begin{pmatrix}10\\30\end{pmatrix}$ from *their* $(10, \textit{their}60)$
(c) (i)	e^{2t} is always positive or oe	B1	
(ii)	$a = 8e^{2t}$ $e^{2t} = \frac{3}{2}$	M1	for attempt to differentiate, must be of the form pe^{2t}, equate to 12 and solve.
	$t = \frac{1}{2}\ln\frac{3}{2}$, $\ln\sqrt{\frac{3}{2}}$ or $\frac{1}{2}\ln 1.5$	A1	Allow fractions equivalent to $\frac{3}{2}$
(iii)	$s = \left[2e^{2t} + 6t\right]_{0.4}^{0.5}$	M1 A1	for attempt to integrate to get $qe^{2t} + 6t$ all correct
	$= (2e + 3) - (2e^{0.8} + 2.4)$	DM1	for correct use of limits or considering distances separately, ignore attempts at *c*
	$(= 8.436 - 6.851)$ $= 1.59$, allow 1.58	A1	